Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

sam.nitk.ac.in nitksam@gmail.com

Theory of Complex Variables - MA 209
Problem Sheet - 3

Polar Form of Complex Numbers

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1. Write the given complex number in polar form first using an argument $\theta \neq Arg(z)$ and then using $\theta = Arg(z)$.

   (a) 2

   (b) $-3i$

   (c) $5-5i$

   (d) $\frac{12}{\sqrt{3}+i}$

2. Use a calculator to write the given complex number in polar form first using an argument $\theta \neq Arg(z)$and then using $\theta = Arg(z)$

   (a) $-\sqrt{2}+\sqrt{7}i$

   (b) $-12-5i$

3. Find z_1z_2 and $\frac{z_1}{z_2}$.Write the number in the form of $a+ib$

   (a) $z_1 = \sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$

   (b) $z_2 = \sqrt{3}(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12}))$

4. Write each complex number in polar form.Finally write the polar form in the form $a+ib$

   (a) $(3-3i)(5+5\sqrt{3}i)$

   (b) $\frac{\sqrt{2}+\sqrt{6}i}{-1+\sqrt{3}i}$

5. Compute the indicated powers.

   (a) $(2-2i)^5$

   (b) $(\sqrt{3}(\cos(\frac{2\pi}{9}) + i\sin(\frac{2\pi}{9}))^6$

6. Write the complex number in polar form and then in the form of $a+ib$
   $\frac{[8(\cos(\frac{3\pi}{8})+i\sin(\frac{3\pi}{8}))]^3}{[2(\cos(\frac{\pi}{16})+i\sin(\frac{\pi}{16}))]^6}$

7. Use De Moivre's formula with $n=2$ to find trigonometric identities for $\cos 2\theta$ and $\sin 2\theta$

8. Use De Moivre's formula with $n=3$ to find trigonometric identities for $\cos 3\theta$ and $\sin 3\theta$

9. Find a positive integer n for which the equality holds.
   $(\frac{\sqrt{3}i}{2} + \frac{1}{2}i)^n = -1$

10. Suppose that $z = r(cos\theta + isin\theta)$. Describe geometrically the effect of multiplying z by a complex number of the form $z_1 = cos\alpha + isin\alpha$ when $\alpha > 0$ and when $\alpha < 0$.

11. Suppose $z = cos\theta + isin\theta$. If n is an integer, evaluate $z^n + \bar{z}^n$and $z^n - \bar{z}^n$.

12. Write an equation that relates $arg(z)$ to $arg(1/z)$, $z \neq 0$.

13. Are there any special cases in which $Arg(z_1z_2) = Arg(z_1) + Arg(z_2)$? Proveyour assertions.

14. How are the complex numbers z_1 and z_2 related if $arg(z_1) = arg(z_2)$?

15. Describe the set of points z in the complex plane that satisfy $arg(z) = \pi/4$.

16. Suppose z_1, z_2, and z_1z_2 are complex numbers in the first quadrant and that the points $z = 0, z = 1, z_1, z_2$, and z_1z_2 are labeled O, A, B, C, and D,respectively. Discuss how the triangles OAB and OCD are related.

17. Suppose $z_1 = r_1(cos\theta_1 + isin\theta_1)$ and $z_2 = r_2(cos\theta_2 + isin\theta_2)$. If $z_1 = z_2$, then how are r_1 and r_2 related? How are θ_1 and θ_2 related?

18. Suppose z_1 is in the first quadrant. For each z_2, discuss the quadrant in which $z_1z_2$could be located.

    (a) $z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ (b) $z_2 = -1$

19. Suppose z_1, z_2, z_3, and z_4 are four distinct complex numbers. Interpret geometrically: $\arg(\frac{z_1-z_2}{z_1-z_2}) = \frac{\pi}{2}$.

20. For $z = neq1$, verify the statement

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}.$$

21. Use part 20 and appropriate results to establish that

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin(n + 1/2)\theta)}{(2\sin(\theta/2)}$$

    for $0 < \theta < 2\pi$. The foregoing result is known as **Lagrange's identity**.

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